Final state interactions for Bose-Einstein correlations in S+Pb $\rightarrow \pi^+\pi^+$ +X reaction at energy 200GeV/nucleon

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February 7, 2008

Abstract

We applied an analytical formula for Bose-Einstein correlations (BEC) developed by us recently to high-energy heavy ion collisions, in particular to data on S+Pb→ π^+-π^++X reaction at energy 200 GeV/nucleon reported by the NA44 Collaboration. It takes into account both Coulomb and strong (π-π s-wave; I=2) final state interactions (FSI). We have found that inclusion of the strong interaction in addition to Coulomb correction affects significantly the extracted parameters of the BEC like the source size R, the degree of coherence λ and the long-range correlation parameter γ. In particular, the λ parameter of the BEC is increased by about 20%. Our results differ from those obtained in e^+e^- annihilation in the following way: the λ parameter does not reach 'chaotic limit' and the γ parameter does not approach to zero.

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Introduction: Recently we have presented an analytical formula for Bose-Einstein correlations (BEC), which takes into account both Coulomb and strong (π-π s-wave; I=2) final state interactions (FSI) and applied it to e^+e^- annihilation data [1]. Our formula has been constructed by using a wave function obtained from applying the phase shift of the strong interaction to the asymptotic Coulomb wave function, and extrapolating it to the non-asymptotic region with the normalization of the Coulomb wave function. We shall apply it here to analysis of high energy nuclear collisions using in addition the following important parameters entering the BEC: degree of coherence λ, long range correlation γ and normalization c. This will make our analysis of the FSI much more clear when compared with a 'standard' fitting formula with or without the Coulomb corrections. In the following sections we first provide brief explanation of our formula, use it next to investigate data for S+Pb$\to$ π^+-π^+ +X collisions at energy 200GeV/nucleon reported by the NA44 Collaboration[2] and, finally, we close our discussion with some concluding remarks.

Coulomb and strong wave function with s-wave phase shift: Let us consider the Coulomb asymptotic wave function for the identical π-π scattering (of relative momentum Q, the pion mass is m_π). Its form is well known and given by [3, 4]:

$$\begin{aligned}\Psi_{\rm C}^{\rm asym}(\mathbf{k},\mathbf{r}) \;&=\; \exp\{i(kz+\eta\ln(k(r-z)))\}\left[1+\frac{\eta^2}{ik(r-z)}\right]\\ &\quad +f(\theta)\frac{\exp\{i(kr-\eta\ln(2kr))\}}{r}\,, \end{aligned}\qquad(1)$$

where $Q=2k,\ z=r\cos\theta$, $\eta=m\alpha/Q$ and the scattering amplitude $f(\theta)$ is given by:

$$f(\theta)=-\frac{\eta}{2k}\frac{1}{\sin^2(\theta/2)}\exp\{-2i\eta\ln\sin(\theta/2)+2i\arg\Gamma(1+i\eta)\}.$$

The s-wave component of the $\Psi_{\rm C}^{\rm asym}(\mathbf{k},\mathbf{r})$ wave function is given by the following formula [3, 4]:

$$\begin{aligned}\Psi_{\rm C(s\text{-}wave)}^{\rm asym}(\mathbf{k},\mathbf{r}) \;&=\; \exp\{i(kr-\eta\ln(2kr)+2\eta_0)\}\frac{1}{2ikr}\left[1+\frac{i\eta(1+i\eta)}{2ikr}\right]\\ &\quad +\exp\{-i(kr-\eta\ln(2kr))\}\frac{1}{-2ikr}\left[1+\frac{i\eta(1-i\eta)}{2ikr}\right]. \end{aligned}\qquad(2)$$

The strong wave fucion of the s-wave has been obtained by extracting the out-going wave (up to the order $\mathcal{O}(1/r)$) of the eq.(2) and using the phase shift of the π-π scattering induced by the strong interaction [5]-[10]. This phase shift is phenomenologically given

by [11]:

$$\delta_0^{(2)} = \frac{1}{2}\left(\frac{a_0 Q}{1+0.5Q^2}\right) \quad (-1.5 \leq a_0 \leq -0.7 \text{ (GeV}^{-1}\text{) }). \tag{3}$$

As a result we obtain the following strong interaction wave function in the asymptotic region:

$$\phi_{\rm st}(\mathbf{k},\mathbf{r}) = f^0(\theta)\frac{\exp\{i(kr-\eta\ln(2kr))\}}{r}, \tag{4}$$

$$f^0(\theta) = \frac{1}{2ik}\exp(2i\eta_0)(\exp(2i\delta_0^{(2)})-1).$$

Therefore, the total asymptotic wave function is expressed as the following sum of both components:

$$\Psi_{\rm total}(\mathbf{k},\mathbf{r}) = \Psi_{\rm C}^{\rm asym}(\mathbf{k},\mathbf{r}) + \phi_{\rm st}(\mathbf{k},\mathbf{r}). \tag{5}$$

In the small kr (internal) region the Coulomb wave function is given by [3, 4]

$$\Psi_{\rm C}(\mathbf{k},\mathbf{r}) \quad = \quad \Gamma(1+i\eta)e^{-\pi\eta/2}e^{i\mathrm{k}\cdot\mathrm{r}}F(-i\eta;1;ikr(1-\cos\theta)), \tag{6}$$

(where F denotes the confluent hypergeometric function) whereas the exact form of the strong wave function in this region is unknown. We assume therefore that the strong wave function is given by the extrapolation of the asymptotic wave function into this internal region [12] with the normalization of the Coulomb wave function [1]:

$$\Psi_{\rm total}(\mathbf{k},\mathbf{r}) = \Psi_{\rm C}(\mathbf{k},\mathbf{r}) + \sqrt{G(\eta)}\phi_{\rm st}(\mathbf{k},\mathbf{r}), \tag{7}$$

where $G(\eta)$ denotes the usual Gamow factor: $G(\eta) = 2\pi\eta/(\exp(2\pi\eta)-1)$ (for the discussion on the validity and usefulness of this assumption and on the smooth connection of wave functions given by eqs.(5) and (7) cf. Ref. [1]). Because in the numerical calculations of the $\Psi_{\rm C}(\mathbf{k},\mathbf{r})$ one encounters a wild oscillating behavior in larger kr region, we use here the 'seamless' fitting method developed in [13].

Theoretical formula of BEC with Coulomb and strong wave function: To describe a pair of the identical bosons, we have to symmetrize its total wave function in the following way:

$$A_{12} = \frac{1}{\sqrt{2}}[\Psi_{\rm C}(\mathbf{k},\mathbf{r}) + \Psi_{\rm C}^S(\mathbf{k},\mathbf{r}) + \Phi_{\rm st}(\mathbf{k},\mathbf{r}) + \Phi_{\rm st}^S(\mathbf{k},\mathbf{r})], \tag{8}$$

where the superscript S denotes the symmetrization of the corresponding wave function. Here $\Phi_{\rm st}(\mathbf{k},\mathbf{r})$ stands for the wave function induced by the strong interactions. Assuming

that a source function is given by $\rho(r)$ we obtain the following expression for the BEC including the FSI [1]:

$$N^{(\pm\pm)}/N^{BG} \quad = \quad \frac{1}{G(\eta)}\int \rho(r)d^3r|A_{12}|^2 \equiv I_{\mathrm{C}} + I_{\mathrm{Cst}} + I_{\mathrm{st}}, \tag{9}$$

$$I_{\mathrm{C}} = \sum_{m,n=0}^{\infty} \frac{1}{m+n+1} I_{R1}(2+m+n)A_1(n)A_1^*(m) \times \left[1 + \frac{n!m!}{(n+m)!}\left(1+\frac{n}{i\eta}\right)\left(1-\frac{m}{i\eta}\right)\right]$$

$$= (1+\Delta_{1\mathrm{C}}) + (E_{2\mathrm{B}} + \Delta_{\mathrm{EC}}),$$

$$I_{\mathrm{C}st} = 2\Re\left[\frac{2}{k}(2k)^{i\eta}\exp\left(-i(\eta_0 + \delta_0^{(2)})\right)\sin\delta_0^{(2)} \sum_{n=0}^{\infty} I_{\mathrm{R2}}(1+n)A_2(n,0)\right],$$

$$I_{\mathrm{st}} = \frac{2}{k^2} I_{\mathrm{R1}}(0)\sin^2\delta_0^{(2)},$$

where

$$E_{2\mathrm{B}} \quad = \quad \int d^3r\ \rho(r)\ e^{-i\mathbf{Q}\cdot\mathbf{r}},$$

$$1+\Delta_{1\mathrm{C}} \quad = \quad 1 + 4\pi\cdot 2\eta \int \rho(r)r^2 dr \sum_{n=0}^{\infty} \frac{(-1)^n A^{2n+1}}{(2n+1)!(2n+1)(2n+2)},$$

$$A_1(n) \quad = \quad \frac{\Gamma(n+i\eta)}{\Gamma(i\eta)}\frac{(-2ik)^n}{(n!)^2},$$

$$A_2(n,l) \quad = \quad \frac{\Gamma(n+l+1+i\eta)\Gamma(2l+2)}{\Gamma(l+1+i\eta)\Gamma(n+2l+2)}\frac{(-2ik)^n}{n!},$$

$$I_{\mathrm{R1}}(n) \quad = \quad 4\pi\int dr r^n \rho(r),$$

$$I_{\mathrm{R2}}(n) \quad = \quad 4\pi\int dr r^{n+i\eta}\rho(r).$$

In this paper $N^{(\pm\pm)}/N^{BG}$ stands for the FSI corrected ratio of pairs of the identical charged bosons measured in a single event to a product of single bosons taken from different events. The $\Delta_{1\mathrm{C}}$ and Δ_{EC} originate from the correction for the finite size effects in the Coulomb interaction. In the following we shall use an artificial Gaussian form of the source function: $\rho(r) = (\frac{1}{\sqrt{2\pi}\beta})^3 \exp(\frac{-r^2}{2\beta^2})$, which results in the following form for its Fourier transform:

$$E_{2\mathrm{B}} = \exp(-\beta^2 Q^2/2).$$

Reanalyses of data reported by NA44 Collaboration: The parameters mentioned at the beginning, namely: degree of coherence λ, long range correlation γ and normalization c, are introduced into our formula eq.(9) in the usual way leading to the following

final formula:

$$N^{(\pm\pm)}/N^{\rm BG}(Q=2k) \;=\; c\,(1+\Delta_{\rm 1C}+\Delta_{\rm EC}+I_{\rm Cst}+I_{\rm st}) \times\left[1+\lambda\frac{E_{\rm 2B}}{1+\Delta_{\rm 1C}+\Delta_{\rm EC}+I_{\rm Cst}+I_{\rm st}}\right](1+\gamma Q)\;. \qquad (10)$$

This formula have been applied to data on S+Pb $\to \pi^+\pi^+$ +X reaction at energy 200 GeV/nucleon reported by the NA44 Collaboration [2].

Before presenting our results based on eq.(10) we must, however, first clarify one point concerning aplication of the Coulomb correction. We show the results in Table I obtained by applying the following pure Coulomb (i.e., without strong interactions) corrected formula:

$$N^{(\pm\pm)}/N^{\rm BG} = c\,(1+\Delta_{\rm 1C}+\Delta_{\rm EC})\times\left[1+\lambda\frac{E_{\rm 2B}}{1+\Delta_{\rm 1C}+\Delta_{\rm EC}}\right](1+\gamma Q), \qquad (11)$$

to the BEC data presented by the NA44 Collaboration. On the other hand, the NA44 Collaboration have also corrected their data by Coulomb correction factors, $K_{Coul}^{\rm NA44}$, which have been calculated by using the Coulomb wave function integration. However, as is shown in the Table I, slight discrepancies are found between their results (obtained by using the $K_{Coul}^{\rm NA44}$) and ours (obtained from minimum χ^2 fit using formula eq.(11)) [1]. The origin of discrepancy shown in the Table I can be traced down to the 15% differences between the $K_{Coul}^{\rm NA44}$ and the $K_{Coul}^{\rm our}$ (see Table II) (one can see this by evaluating the correction factor, $K_{Coul}^{\rm our} = 1/G(\eta)[1+\Delta_{\rm 1C}+\Delta_{\rm EC}]$). Our final results obtained by using eq.(10) are shown in Tables III, IV and in Fig.1. For the sake of reference the results of eq.(11) and of the 'standard' fitting formula:

$$N^{(\pm\pm)}/N^{\rm BG} = c\,[1+\lambda E_{\rm 2B}]\,(1+\gamma Q), \qquad (12)$$

are also shown there. It is found that the FSI corrections, which take into account not only the Coulomb but also the strong interactions between the identical charged pions, affect substantially the source size parameter R, the degree of coherence parameter λ and the long-range correlation parameter γ. When compared with the Coulomb correction case only (i.e., with eq.(11)), the R parameter is reduced by about 5% and the λ parameter is increased by about 20% remaining, however, always below its 'chaotic' limit value

[1]To compare our estimated values with those reported by the the NA44 Collaboration, we have to use a relation, $\beta = \sqrt{2}R$ (which reflects the fact that NA44 Collaboration employed the following fitting formula: $c\,(1+\lambda\exp(-Q^2R^2))$).

of $\lambda = 1$[1] (notice that in e^+e^- annihilation the corrections caused by the FSI pushes the λ towards this limit). In what concerns the parameter γ, we have found that it does not reduce to zero (differently than in the e^+e^- annihilation case).

Concluding remarks: We applied our improved formula for the BEC (which accounts for the FSI) to data for S+Pb $\rightarrow \pi^+\pi^+$ +X at energy 200GeV/nucleon presented recently by the NA44 Collaboration. We have found that inclusion of the full FSI (including also the strong interactions, cf eq.(10)) increases the extracted value of the degree of coherence parameter λ by about 20% in comparison to the case where only the Coulomb correction are applied (cf. eq.(11)). However, differently than in e^+e^- annihilation case (cf. [1]) it does not reach 'chaotic' limit of $\lambda = 1$. We have also found that the long-range correlation parameter γ does not approach to zero which also seems different from the e^+e^- annihilation case. On the other hand the source size parameter R is decreased by about 5% only.

Acknowlegements:
The authors would like to thank T.Mizoguchi for his kind correspondence. One of authors (T.O.) would like to thank Shinshu Univ. for their hospitalities. This work is partially supported by Japanese Grant-in-Aid for Scientific Research from the Ministry of Education, Science Sport and Culture (#06640383).

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Figure Captions.

Fig. 1. Analysis of data for S+Pb→ $\pi^+\pi^+$ +X reaction at energy 200 GeV/nucleon reported by the NA44 Collaboration[2] by the minimum χ^2 fitting method. The strong interaction phase shift parameter has been set to $a_0 = -1.20$ GeV^{-1}. The solid and dotted curves represent fitted result obtained by using eq.(10) with and without factor (1+γQ), respectively.

Table Captions.

Table 1. Results of analysis of BEC data for S+Pb→ $\pi^+\pi^+$ +X reaction at energy 200 GeV/nucleon reported by the NA44 Collaboration[2]. Comparison of our results obtained from fitting of the eq.(11) with values reported by the NA44 Collaboration which were obtained by using the Coulomb correction factor K_{Coul}^{NA44}[2].

Table 2. Comparison of the Coulomb correction factor K_{Coul}^{NA44} (used in ref.[2]) with the K_{Coul}^{our} (evaluated from eq.(2)).

Table 3. Results of analysis of BEC data for S+Pb→ $\pi^+\pi^+$ +X reaction at energy 200 GeV/nucleon reported by the NA44 Collaboration[2]. Comparison of our results obtained from fitting of eqs.(10), (11) and (12) without factor $(1+\gamma Q)$. We have employed the parameter $a_0 = -1.20$ GeV^{-1}.

Table 4. Results of analysis of BEC data for S+Pb→ $\pi^+\pi^+$ +X reaction at energy 200 GeV/nucleon reported by the NA44 Collaboration[2]. Comparison of our results obtained from fitting of the eqs.(10), (11) and (12). We have employed the parameter $a_0 = -1.20$ GeV^{-1}.

Table I:

formula	c	R [fm]	λ	γ	χ^2/NDF
correction by K_{Coul}^{NA44}	1.000±0.002	4.50±0.431	0.46±0.04	—	18.1/16
eq. (11) without $(1+\gamma Q)$	1.000±0.003	4.697±0.272	0.481±0.031	—	34.5/16
eq. (11)	1.031±0.009	5.159±0.331	0.453±0.034	−0.181±0.048	22.1/15

Table II:

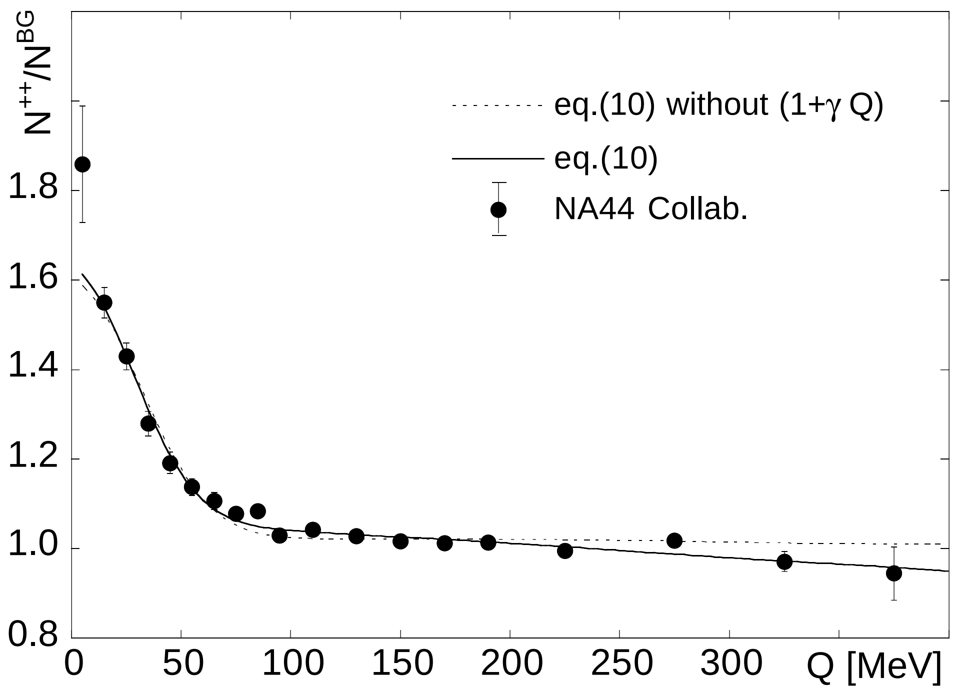
Q [MeV/c]	K_{Coul}^{NA44}	K_{Coul}^{our}
5	1.565	1.807
15	1.161	1.112
25	1.074	1.037
35	1.037	1.014
45	1.020	1.008
55	1.012	1.006
65	1.009	1.006
75	1.007	1.006
85	1.005	1.005
95	1.005	1.004
110	1.003	1.003
130	1.003	1.002
150	1.002	1.000
170	1.002	0.999
190	1.001	0.999
225	1.001	1.000
275	1.001	1.003
325	1.001	1.004
375	1.000	1.007

Table III:

formula	c	R [fm]	λ	γ	χ^2/NDF
eq. (10) strong + Coulomb	1.007±0.003	4.422±0.282	0.593±0.028	—	46.5/16
eq. (11) Coulomb	1.000±0.003	4.697±0.272	0.481±0.031	—	34.5/16
eq. (12) standard	1.019±0.003	4.209±0.206	0.528±0.031	—	48.1/16

Table IV:

formula	c	R [fm]	λ	γ	χ^2/NDF
eq. (10) strong + Coulomb	1.045±0.008	4.870±0.320	0.541±0.031	-0.232±0.047	24.6/15
eq. (11) Coulomb	1.031±0.009	5.169±0.331	0.453±0.034	-0.181±0.048	22.1/15
eq. (12) standard	1.069±0.010	4.929±0.292	0.514±0.034	-0.274±0.051	18.1/15



N^{++}/N^{BG}
eq.(10) without (1+γ Q)
eq.(10)
NA44 Collab.
1.8
1.6
1.4
1.2
1.0
0.8
0
50
100
150
200
250
300
Q [MeV]

Fig.1